AMATH 350 Final Exam Review

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Theorems With Proofs

Theorem 0.1. *(Existence and Uniqueness of 1st Order Solutions) The initial value problem* $\frac{dy}{dx} = f(x,y), y(x_0) = y_0$ *has a unique solution defined on some interval around* x_0 *of* $f(x,y)$ *and* $f_y(x,y)$ *are continuous within some rectangle containing the point* (x_0, y_0)*.*

Theorem 0.2. *(Existence and Uniqueness Theorem) Consider the IVP of (1) and the* n *initial conditions*

$$y(x_0) = p_0, y'(x_0) = p_1, ..., y^{(n-1)}(x_0) = p_{n-1}$$

Then there exists a unique solution if there is an open interval I *containing* x_0 *such that*
1. The functions $a_n(x), a_{n-1}(x), ..., a_0(x), F(x)$ *are continuous*
2. $a_n(x) \neq 0$ *on* I
(Alternatively, if we put the equation in standard form

$$y^{(n)}(x) + b_{n-1}(x)y^{(n-1)}(x) + ... + b_0(x)y(x) = G(x)$$

then we just need $b_0, ..., b_{n-1}, G$ *to be continuous)*

Theorem 0.3. *(Principle of Superposition [I]) Let* Φ *be a linear differential operator. If* y_1 *is solution to* $\Phi(y) = F_1(x)$ *and* y_2 *is a solution to* $\Phi(y) = F_2(x)$ *then* $y_1 + y_2$ *is a solution to* $\Phi(y) = F_1(x) + F_2(x)$*.*

Corollary 0.1. *If* y_h *is a solution to* $\Phi(y) = 0$ *and a solution* y_p *to* $\Phi(y) = F(x)$*, then* $y_h + y_p$ *is also a solution to* $\Phi(y) = F(x)$*.*

Theorem 0.4. *(Principle of Superposition [II]) If* y_1 *and* y_2 *are both solutions to* $\Phi(y) = 0$*, then so is* $y = c_1y_1 + c_2y_2$ *for any* $c_1, c_2 \in \mathbb{R}$*.*

Theorem 0.5. *If* $W(x_0) \neq 0$ *for some* $x_0 \in I$ *then* $f_1, f_2, ..., f_n$ *are linearly independent on* I*.*

Remark 0.1. In general, the converse of the above statement is not true. A famous counterexample is $f(x) = x^2|x|$ and $g(x) = x^3$. You can show that $W(f,g) = 0$ but f and g are clearly independent. However, we can add one more condition to make this true.

Theorem 0.6. *Let* $p(x)$ *and* $q(x)$ *be continuous on an interval* I *and suppose that* $y_1(x)$ *and* $y_2(x)$ *are solutions to the homogeneous linear equation*

$$y'' + p(x)y' + q(x)y = 0$$

on I*. If* $W(y_1, y_2) = 0$ *for some* $x_0 \in I$*, then* y_1 *and* y_2 *are linearly dependent.*

Proposition 0.1. *(Abel's Formula) If* y_1 *and* y_2 *are solutions to* $y'' + p(x)y' + q(x)y = 0$ *then*

$$W(y_1, y_2) = W(x_0)e^{-\int_{x_0}^{x} p(x)\ dx}$$

Theorems Without Proofs

See the review sheet for these proofs:

- Classification of PDEs (Hyperbolic, Parabolic, Elliptic)
- Shifting Property of the Fourier Transform
- Convolution Theorem of the Fourier Transform